

Similar Part Rearrangement in Clutter With Pebble Graphs

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Abstract—This work proposes a method for effectively computing manipulation paths to rearrange similar objects in a cluttered space. The solution can be used to place similar products in a factory floor in a desirable arrangement or for retrieving a particular object from a shelf blocked by similarly sized objects. These are challenging problems as they involve combinatorially large, continuous configuration spaces due to the presence of multiple moving bodies and kinematically complex manipulators. This work leverages ideas from algorithmic theory, multi-robot motion planning and manipulation planning to propose appropriate graphical representations for this challenge. These representations allow to quickly reason whether manipulation paths allow the transition between entire sets of objects arrangements without having to explicitly enumerate the path for each pair of arrangements. The proposed method also allows to take advantage of precomputation given a manipulation roadmap for transferring a single object in the same cluttered space. The resulting approach is evaluated in simulation for a realistic model of a Baxter robot and executed in open-loop on the real system, showing that the approach solves complex instances and is promising in terms of scalability and success ratio.

I. INTRODUCTION

Robot manipulators can benefit from the ability to rearrange objects in constrained, cluttered human environments. Such a skill can be useful, for instance, in manufacturing, where multiple products need to be arranged in an orderly manner or in service robotics where a robotic assistant, in order to retrieve a refreshment from a refrigerator, must first rearrange other items. This paper proposes a methodology for solving such tasks in geometrically complex and constrained scenes using a robotic arm. The focus is on the case that the target objects are geometrically similar and interchangeable.

A key challenge in developing practical algorithms for rearrangement challenges is the size of the search space. A complete method must operate in the Cartesian product of the configuration spaces of all the objects and the robot. Problems also become hard when the objects are placed in tight spaces, coupled with limited manipulator maneuverability. This paper deals primarily with these combinatorial and geometric aspects and proposes motion planning methods that return collision-free paths for manipulating rigid bodies. Several issues also arise in real-world applications, which are studied here, such as accurate estimation of object locations from sensing data.

The approach reduces the continuous space, high-dimensional rearrangement problem into several, discrete challenges on “rearrangement pebble graphs” (RPGs). The inspiration comes from work in algorithmic theory on “pebble

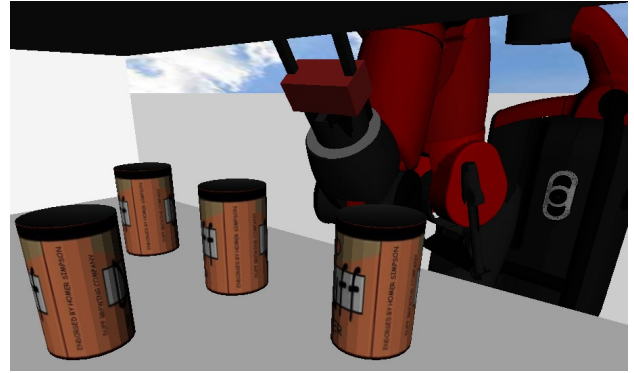


Fig. 1. In order for Baxter to grasp the can at the back of the shelf, the other cans need to be rearranged.

graphs” [4], recent contributions in multi-robot motion planning [38], as well as work in manipulation planning [37].

The RPGs contain stable poses for the objects as nodes. Edges indicate that there is a path for which the manipulator can transfer an object between the corresponding poses, even if all other poses are occupied. For most interesting queries, it is difficult to find a single pebble graph that contains both the start and the goal object arrangements and solves the problem. Instead, it is necessary to generate multiple such graphs and then identify transitions between them to find a solution. RPGs can implicitly represent an entire set of object arrangements over their nodes. Given that the objects are similar and unlabeled, objects can be safely moved along an edge of an RPG. Only once a sequence of pebble graph transitions that solves the problem has been found does the algorithm need to extract an explicit sequence of object placements and the corresponding manipulation paths. This encapsulation of multiple object arrangements within an individual graph helps with the combinatorial aspects of rearrangement planning. Furthermore, the method can effectively utilize precomputation. For a known workspace and geometry for the objects, it is possible to precompute a manipulation graph and perform many expensive collision checking operations offline.

Relative to the state-of-the-art, the approach does not drastically limit the type of rearrangement challenges that can be addressed to achieve efficiency. Nevertheless, it does make certain concessions. For instance, it must be possible to retract the arm to a safe configuration from every stable grasped pose in a solution sequence. Given this requirement for solutions,

probabilistic completeness can be argued within this set while also achieving computational efficiency, the same way that it was achieved for multi-robot path planning [38]. Furthermore, the relation to this previous work implies a way to extend the proposed framework to dissimilar objects. Nevertheless, the transfer of the idea of pebble graphs to the problem of rearrangement was not trivial. The presence of a manipulator in rearrangement planning induces additional constraints relative to the previous work on multi-robot motion planning [38], which required the development of different solutions for the connection of RPGs.

II. RELATED WORK AND CONTRIBUTION

Rearrangement planning [5, 36] has been approached from many different perspectives in the robotics literature.

Planning among Movable Obstacles: A related challenge for mobile systems is the problem of navigation among movable obstacles (NAMO). Early on it was shown that NAMO is NP-hard [47] and later on it was confirmed that this is the case even for simpler instances that involve only unit square obstacles [16]. Due to the problem’s complexity, most efforts have focused on efficiency [12, 35] and provide completeness only in certain cases [39, 40]. A probabilistically complete solution for NAMO was eventually provided [44], but it can only be applied to lower dimensional robots (2-3 DOFS) and corresponds to uninformed brute force search. More recently, a decision-theoretic framework for NAMO has also been presented, which deals with the inference uncertainty in both perception and control of real robots [32].

An interesting related challenge is the minimum constraint removal problem, where the goal is to minimize the amount by which constraints must be displaced to get a feasible path [20]. For this problem any asymptotically optimal solution has recently been achieved [21] but the problem does not consider negative interactions between obstacles as in the current setup.

Manipulation Planning and Grasping: The focus in this work is on building solutions for high-DOF robotic arms and solve manipulation challenges. Such problems can be approached with a multi-modal “manipulation graph” abstraction that contains “transit” and “transfer” paths [1, 2, 37] and which can be constructed through a sampling-based process [27, 30]. This paper follows a similar formalization and applies it appropriately to the case of multiple similar movable objects towards achieving an efficient solution. The current work is also utilizing asymptotically optimal sampling-based planners for the computation of the manipulation graph [26]. Tree-based sampling-based planners have also been used successfully in the context of manipulation planning [7, 8].

There is also a variety of different approaches for manipulation planning, which employ heuristic search [14] or optimization methodologies, such as CHOMP [50, 28]. The focus of the current paper is more on the combinatorial challenges that arise from reasoning about multiple objects and not the actual manipulation method. In this process, this work is making significant use of heuristic search primitives over sampling-based roadmaps. The output of the current algorithm

could also potentially be integrated with methods like CHOMP to improve the quality of the computed solution. There is also significant effort that focuses on identifying appropriate grasps in complex scenes [6, 13] but the current paper does not focus on this aspect of the challenge.

The above manipulation efforts deal with moving and grasping individual objects. Manipulation planning among multiple movable obstacles has been considered before for “monotone” problems where each obstacle can be moved at most once [41, 34]. The solution in the current paper can reason about more complex challenges. Assembly planning is also solving similar multi-body problems but the focus there is on separating a collection of parts and typically the robot path is ignored [48, 19, 42]. Another paradigm for dealing with cluttered scenes involves non-prehensile manipulation, such as pushing objects [15, 17]. The current solution could potentially be extended to include such actions but the focus in this paper is on grasping primitives.

Task and Motion Planning: Rearrangement planning is an instance of integrated task and motion planning, which can be seen as an important step towards solving complex challenges in robotics and especially in manipulation [11, 23, 25]. Such integrated problems have been approached through multi-modal roadmap structures before to deal with the presence of both discrete and continuous parameters [9, 22, 23], which is also useful in the current challenge.

Multi-robot Motion Planning: A major motivation for the current work is to utilize recent progress in multi-robot motion planning solvers [45, 33, 46, 49] in the context of manipulation planning. Multi-robot motion planning is itself a hard problem due to the high-dimensionality introduced from the presence of multiple moving bodies. Thus, coupled, complete approaches typically do not scale well with an increased number of robots, although there are efforts in decreasing the number of effective degrees of freedom [3]. On the other hand, decoupled methods, such as priority-based schemes [43] or velocity tuning [31], trade completeness for efficiency.

An interesting line of work in algorithmic theory focuses on “pebble motion on graphs”, where distinct pebbles need to move from an initial to a goal vertex assignment on a graph [29]. The unlabeled version of the problem has also been studied in the literature [10]. It turns out that testing the feasibility of a “pebble motion” problem can be answered in linear time [4, 18]. These results from algorithmic theory, integrated with sampling-based algorithms for motion planning, inspired a recent method for in continuous representations [38]. This approach reduces the multi-robot motion planning problem into a sequence of discrete pebble problems in a manner that is possible for the algorithm to efficiently transform movements of pebbles into valid motions of the robots.

The current paper is motivated by the progress achieved by this recent multi-robot motion planning work and the results in algorithmic theory. It aims to take advantage of similar primitives from the “pebble motion” problem to solve in rearrangement planning challenges where objects motions have to also respect manipulation constraints.

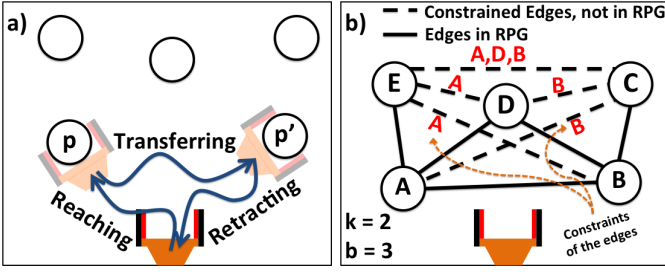


Fig. 2. Each edge on an RPG is the combination of a reaching, transferring and retracting path.

III. PROBLEM SETUP AND NOTATION

Consider a **workspace** $\mathcal{W} \subset \mathbb{R}^3$ with static obstacles, a set of k movable unlabeled rigid body objects with the same geometry, and a manipulator \mathcal{M} . The subset of \mathcal{W} not occupied by static obstacles is defined as \mathcal{W}^f . Each **object** \mathcal{O}_i can acquire a **pose** $p_i \in \mathcal{P}$ and for that pose its geometry occupies a subset of the workspace. The poses are general 3D transformations, i.e., $\mathcal{P} \subseteq SE(3)$. For *collision-free* poses $p_i^f \in \mathcal{P}^f$, the interior of the objects are placed in the collision-free workspace. This definition allows the objects to touch obstacles.

An **unlabeled arrangement** $\alpha \in \mathcal{A}$ specifies a k -combination of poses $\{p_1, \dots, p_k\}$, where $p_i \in \mathcal{P}$ and results in a placement of the objects in \mathcal{W} . Permutations of α are indistinguishable, since the objects are unlabeled. A *collision-free arrangement* $\alpha^f \in \mathcal{A}^f$ does not cause collisions among objects, as well as between obstacles and objects. Similarly define **configurations** $q^M \in \mathcal{Q}^M$ for the **manipulator** \mathcal{M} . *Collision-free configurations* q^f place the manipulator in the collision-free workspace \mathcal{W}^f .

The **configuration space** $\mathcal{Q} = \mathcal{Q}^M \times \mathcal{A}$ corresponds to the Cartesian product of the manipulator's c-space and the space of unlabeled object arrangements. The *collision-free configuration space* \mathcal{Q}^f does not allow collisions between the manipulator or the objects with obstacles, as well as between objects, or between the manipulator and objects. While penetrations are not allowed between objects and the robot, the arm can grasp the objects.

The manipulation problem has then two types of valid configurations: (a) **Stable configurations** $q^s \in \mathcal{Q}^s$: Collision-free configurations where the objects are in stable poses where they rest without any forces from the arm; (b) **Grasping configurations** $q^g \in \mathcal{Q}^g$: These correspond to configurations where the manipulator is grasping an object. Let $\mathcal{Q}^v = \mathcal{Q}^s \cup \mathcal{Q}^g$ denote the *valid configuration space* for the rearrangement problem. The simulations for this paper use as the stable set of poses $\mathcal{P}^s \subset \mathcal{P}$ the placement of the objects on horizontal surfaces, i.e., $\mathcal{P}^s \equiv SE(2)$.

Furthermore, define a **type** $t \in \mathbb{T}$ for such manipulation problems (also called a “mode”), which has two values. In the *transit* type, the manipulator is not carrying an object and $q \in \mathcal{Q}^s$. In the *transfer* type, the manipulator carries an object and $q \in \mathcal{Q}^g$. The only way that the type can change

is if $q \in \mathcal{Q}^t = \mathcal{Q}^s \cap \mathcal{Q}^g$. Such a configuration q is called a *transition* configuration. The problem type can change only if the configuration of the problem is a transition one.

The problem's **state space** $\mathbb{X} : \mathcal{Q} \times \mathbb{T}$ arises from the combination of the configuration space \mathcal{Q} and the type space \mathbb{T} . It is then easy to define the sets \mathbb{X}^f (free), \mathbb{X}^s (stable), \mathbb{X}^g (grasping) and \mathbb{X}^t (transition), e.g., $(\mathbb{X}^t = \mathcal{Q}^t \times \mathbb{T})$.

The Unlabeled Rearrangement Problem: Given an initial state $x_0 = ((q_0, \alpha_0), t_0) \in \mathbb{X}^v$ and a final state $x_1 = ((q_1, \alpha_1), t_1) \in \mathbb{X}^v$, compute a continuous *path* $\pi \in \Pi : [0, 1] \rightarrow \mathbb{X}^v$ such that $\pi(0) = x_0$, $\pi(1) = x_1$ and the types of states along path π change only when $\pi(s) \in \mathbb{X}^t$.

Thus, a path is an alternating sequence of transit and transfer states, which change at transition configurations. Additional definitions and notation are introduced as needed during the description of the proposed solution.

IV. REARRANGEMENT PLANNING

One way of solving the unlabeled rearrangement problem would be to build a manipulation graph [2] in the entire \mathbb{X} . This is, however, a high-dimensional space and given that motion planning is hard, efficient solutions cannot be achieved easily as the number of objects increases. The idea here is to abstract out the motion of the manipulator and then reason directly about the movement of objects between different stable poses in \mathcal{P}^s . Reasoning about the movement of multiple objects can take place over discrete graphical representations so as to take advantage of linear-time path planning tools for rearranging unlabeled “pebbles” on a graph from an initial to a target arrangement [4].

A sampling approach can be used to define graphs where nodes correspond to stable poses in \mathcal{P}^s and edges correspond to collision free motions of the arm that transfer an object between stable poses. If such a graph is connected and contains all the poses from the initial and target arrangements, then a discrete solver can be used to define a solution in the continuous space as long as placing objects in different poses does not cause collisions [4].

It may be difficult or impossible, however, to construct a single such graph that directly solves the problem. For example, the poses in the initial and target arrangements could be already overlapping, or it may not be possible to ensure connectivity with collision-free motions of the arm. Motivated by work in the multi-robot motion planning literature [38], the current paper considers multiple such graphs, referred to “rearrangement pebble graphs” (RPGs).

Within each RPG the discrete solver can be used to achieve all feasible arrangements, given its connectivity. If the RPG has one connected component, then all possible arrangements over the graph can be attained for unlabeled objects and they do not need to be explicitly stored. If the RPG has multiple connected components, a signature, which specifies how many objects exist in each connected component, is sufficient to describe the feasible arrangements. It should also be possible to switch between different RPGs, if they share at least k poses that can be occupied by objects, given the corresponding signatures.

This gives rise to a hyper-graph structure, where each node corresponds to an RPG and a signature. Edges correspond to transitions between such hyper-nodes. The initial and target arrangements define two such hyper-nodes. Then the approach generates and connects hyper-nodes until the initial and target arrangement are connected on the hyper-graph. At that point, the rearrangement problem is solved and the necessary motions of the manipulator can be extracted along the path connecting the initial and target nodes on the hyper-graph.

A. Rearrangement Pebble Graphs

RPGs are constructed so that the objects are placed in collision-free, stable poses. They contain at least k poses as nodes and b additional nodes. The extra nodes allow the rearrangement of k objects on the graphs. The set of poses used in an RPG is defined as a pumped arrangement:

Definition: A pumped arrangement α^P is a set of $n = k + b$ poses where: (a) These poses are a subset of collision-free, stable poses \mathcal{P}^s . (b) No two objects would collide if they were placed on any two poses of the pumped arrangement.

Definition: An RPG is a graph $\mathcal{G}_P(\alpha^P, \mathcal{E}^P)$ where the set of nodes corresponds to a pumped arrangement α^P . The set of edges \mathcal{E}^P corresponds to pairs $p, p' \in \alpha^P$, for which the manipulator can transfer objects between poses p, p' without collisions given that potentially every other pose of α^P is occupied.

Algorithm 1: CREATE_RPG(q_s^M, k, b)

```

1  $V_P \leftarrow \text{Sample\_Valid\_Pumped\_Arrangement}(k + b)$  ;
  // sample  $k + b$  poses
2  $E_P \leftarrow \emptyset, E_c \leftarrow \emptyset$  ;           // initialize data
  structure
3 for  $p, p' \in V_P$  and  $p \neq p'$  do
4    $\pi \leftarrow \text{Compute\_Minimum\_Conflict\_Path}(q_s^M, p, p')$ ;
5   if  $\text{is\_valid}(\pi)$  then
6      $E_P \leftarrow E_P \cup ((p, p'), \pi)$  ; // edge is added
      to the RPG
7   else
8      $c \leftarrow \text{Compute\_Constraints}(\pi)$  ; // find
      the constraining poses
9      $E_c \leftarrow E_c \cup ((p, p'), \pi, c)$  ; // store
      constrained edge in  $E_c$ 
10 return  $\{\mathcal{G}_P(V_P, E_P), E_c\}$ 

```

Algorithm 1 describes the construction of an RPG. The algorithm receives as input a “safe” configuration of the manipulator q_s^M , which is a configuration that does not interfere with the objects placed on any stable poses and typically is a retracted arm configuration. The algorithm needs also the size of the RPG, i.e., the parameters k and b . The algorithm will return an RPG and a set of edges E_c that were not added to the RPG together with the poses that blocked them.

The algorithm starts by selecting a non-intersecting set of $k + b$ poses that create a pumped arrangement (Line 1). For

each pair of poses p and p' a path has to be computed that allows the manipulator to move an object from p to p' . This path consists of three segments as shown in Fig. 2:

- i) A path for the arm from q_s^M to a transition state, $x_p^t \in \mathbb{X}^t$, for pose p ,
- ii) A path that transfers the object from $x_p^t \in \mathbb{X}^t$ to a state $x_{p'}^t \in \mathbb{X}^t$ for p' ,
- iii) A retraction path from $x_{p'}^t$ back to q_s^M .

For each segment the algorithm aims to compute a “minimum conflict path” given that objects may be placed in all poses but p and p' . If there is a collision-free path, then an edge (p, p') is added to the RPG (Line 6). The path used to validate the transition is also stored on the edge. In case the “minimum conflict path” π collides with objects on other poses of the RPG, the edge (p, p') is not added. The algorithm identifies these potential collisions and stores them in a set of constraints c (Line 8). Edges with constraints are stored in the data structure E_c (Line 9). The algorithm returns the RPG and the set E_c of constrained edges. The use of E_c is described later on.

B. Constructing Hypernodes

Objects within each connected component can be rearranged, since the edges within an RPG correspond to valid motions, regardless of object placement. The number of objects actually in each connected component is tracked by a signature.

Definition: A signature, $\sigma_{\mathcal{G}_P}(\alpha)$ is the number of objects contained in each connected component of RPG \mathcal{G}_P according to an arrangement α .

The important observation is that all arrangements in an RPG which have the same signature are reachable one from another, for unlabeled objects. That is, if $\sigma_{\mathcal{G}_P}(\alpha) = \sigma_{\mathcal{G}_P}(\alpha')$, then there is some sequence of transitions and a corresponding path for the manipulator, π , which brings α to α' , and vice versa.

Definition: A hyper-node is an RPG and a signature σ shared by a set of reachable arrangements of objects on the RPG.

Algorithm 2 takes as an argument the existing hyper-graph and adds new hyper-nodes and edges. It needs to be aware of the safe configuration q_s^M , and the size of the RPGs it will construct: $n = k + b$. When this function completes, the new nodes will be in the hyper-graph and appropriately connected with edges.

The algorithm begins by constructing a random, valid RPG (Line 1). Then, it computes all possible signatures that the generated RPG can attain (Line 2). A new hyper-node v_H is created for each signature (Line 5). When v_H is added in the hyper-graph H , function `CONNECT_NODE` tries to connect v_H with the already existing nodes in H corresponding to different RPGs (Line 8). Finally, when all the “siblings”, nodes that correspond to the same RPG have been created, function `CONNECT_SIBLINGS` will try to add edges between them (Line 9).

C. Connecting Hypernodes

The hyper-nodes in H have two different ways to connect to each other: (A) `CONNECT_NODES` connects nodes from a

Algorithm 2: CREATE_HYPERNODES($H(V_H, E_H), q_s^M, k, b$)

```

1  $\{\mathcal{G}_P, E_c\} \leftarrow \text{CREATE\_RPG}(q_s^M, k, b);$  // create an RPG
2  $\Sigma \leftarrow \text{Generate\_Signatures}(\mathcal{G}_P);$  // compute all signatures
3  $V_n \leftarrow \emptyset;$  // initialize "set of sibling RPGs
4 for  $\sigma \in \Sigma$  do
5    $v_H \leftarrow (\mathcal{G}_P, \sigma);$  // construct a new hyper-node
6    $V_n \leftarrow V_n \cup v_H;$  // keep the new node as a "sibling" node
7    $V_H \leftarrow V_H \cup v_H;$  // add the new node to the hyper-graph
8    $\text{CONNECT\_NODE}(H, v_H);$  // connect new node to non-siblings
9  $\text{CONNECT\_SIBLINGS}(H, V_n, E_c);$  // connect "siblings"

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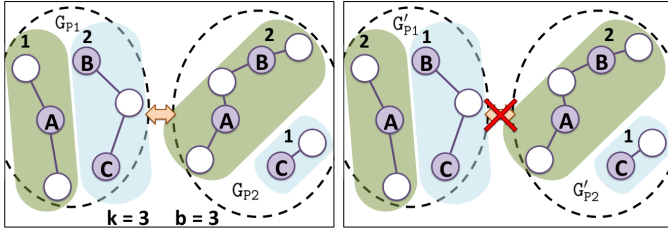


Fig. 3. An illustration of **CONNECT_NODE** that both RPGs share the shaded positions (A,B,C). (Left) A connection is made between the nodes, as the signature information allows all three common positions to contain an object in both RPGs. (Right) The two nodes cannot be connected, because on \mathcal{G}'_{P1} it is not possible to place objects on both common poses B and C.

new RPG with nodes from previous RPGs, based on similar poses and signatures. The function first identifies whether node v_H shares at least k common poses with an existing node. If true, the method checks whether both nodes can achieve placing k objects on the k common poses given the nodes' signatures (Fig. 3). If they both can, an edge is added between the two nodes, which represents a context switch between two arrangement sub-problems. (B) **CONNECT_SIBLINGS** connects nodes that come from the same RPG by using the constraint edges E_c , to switch between different signatures on the same RPG. The algorithm connects nodes only if there is a motion that allows transition between signatures of the same RPG (Fig. 4).

The arm is able to follow a constrained edge in E_c if the constraining poses can be emptied given the hyper-node's signature. For all the constrained edges the algorithm finds the connecting components of the RPG that this edge is connecting. If the connecting components are different and the edge is feasible given the signature, then this edge can be used for potential connections between two hyper-nodes. An edge is feasible if the poses that constrain the edge can

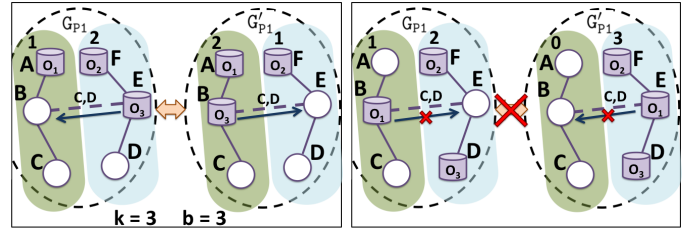


Fig. 4. **CONNECT_SIBLINGS**: Nodes from the same RPG but different signatures. The constrained (dashed) edge is not part of the RPG but can be used to change signature. (Left) The nodes can be connected as moving an object along the edge is feasible given the constraints (C,D). (Right) The two nodes cannot be connected, because the pose D cannot be emptied.

be emptied given the signature of v_H and the target pose p' of the constrained edge can be emptied. Furthermore, it must be possible to bring an object to the source pose p of the edge. Moving along such edges results in a change of signature relative to that of v_H and the new signature σ_n can be computed. Then an edge is created that connects the two "sibling" nodes in the hyper-graph.

D. Creating The Hypergraph

The overall approach operates similarly to a bidirectional tree sampling-based planner (e.g., EST) [24]. It first generates two hyper-nodes of size k for the initial and target arrangements. To help the generation of edges between hyper-nodes, a seed arrangement, α_{seed} , is used to construct the pebble graph for new nodes in Alg. 2. It selects an existing node on the tree randomly and a seed α_{seed} of k poses are selected that agree with the signature of the selected node. This seed is used inside Alg.2 to generate the new RPG and the corresponding hyper-nodes. The method keeps doing this until the initial and goal hyper-nodes are in the same connected component. Upon completion, the method generates the sequence of motion plans from the hyper-graph that satisfies the query.

E. Answering Queries

For each hyper-node, there exists a way to move between all possible arrangements of objects on the corresponding RPG that have the same signature. But these actual paths that accomplish these rearrangement are not explicitly stored. Only during the query phase, when a graph search returns a solution sequence of hyper-nodes and edges the algorithm computes solutions to the pebble motion problem within each RPG.

The hyper-graph path is transformed into a manipulation path by solving discrete graph problems [4] on individual RPGs given start and final arrangements on them according to information stored on the hyper-edges. The algorithm requires a start and a final arrangement on the RPG. Because the RPG contains edges which have safe motions for the manipulator, any arrangement produced by the approach will be feasible for the robotic arm if it follows the sequence of actions that was used to validate the edge. The start arrangement on the RPG is simply the latest arrangement the objects have reached in the previous RPG. The end arrangement is stored on the edges of the hyper-graph. For connections between nodes generated

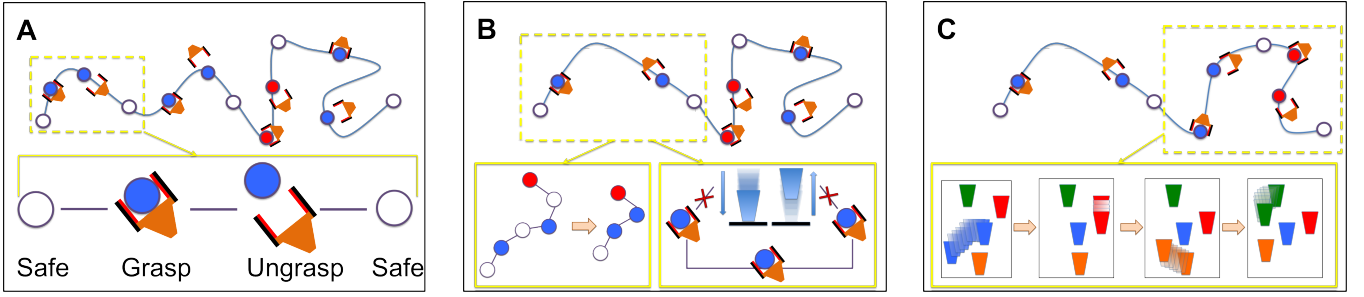


Fig. 5. A solution trajectory can be decomposed into intervals of reaching, transferring and retracting as in A. If the same grasp on the same object exists consecutively in the solution, then the intermediate safe state is removed. In addition, any symmetrical sequence of states, such as lowering and raising the object, can also be removed, as in B. In order to remove redundant intermediate placements of the same object, placement of the same object in different parts of the trajectory are checked if they can be removed and the object directly placed the the next position.

from the same RPG, the edges involve a motion and have the constraints associated with this motion encoded. The end configuration is such that those constraints are satisfied.

F. Smoothing Solutions

Figure 5 shows the operation of smoothing. The colored disks represent poses occupied by an object, while the uncolored discs represent the safe configuration q_s^M . The edges connecting the nodes represent the trajectory taken by the manipulator to move the objects. A trajectory can be separated into distinct sequences of transit to grasp from q_s^M , transfer an object between poses (p, p') , and retract to q_s^M , as shown in Figure 5a.

Phase 1, Consecutive, Identical Grasps: For each of these distinct sequences, the smoothing process first looks for consecutive, identical grasps on the same object. If these consecutive sequences exist, any intermediate safe state is removed, along with any redundant states, as shown in Figure 5b. This has the effect of removing motions such as raising and lowering the object, which is an unnecessary motion when consecutively grasping the same object.

Phase 2, Standard Trajectory Smoothing: Each reaching, transferring and retracting sequence is smoothed by checking for shortcuts between pairs of states. If these shortcuts exist, they replace their corresponding intermediate trajectory.

Phase 3, Maximizing Consecutive Grasps: The solution returned by the algorithm is conservative. This can potentially lead to redundant movements of objects. The trajectory that moves an object o_x from position p to p' can be represented as $o_x^p \rightarrow o_x^{p'}$. The trajectory sequence $o_a^i \rightarrow o_a^{i+1} \rightarrow o_b^j \rightarrow o_b^{j+1} \rightarrow o_a^{i+1} \rightarrow o_a^{i+2}$ contains an intermediate placement of o_a at pose $i+1$, before eventually moving it to $i+2$ (Figure 5c). If (I) o_a can be moved from pose i to pose $i+2$ with a collision free path, given that o_b is at pose j and (II) the trajectory of $o_b^j \rightarrow o_b^{j+1}$ is collision free with o_a at pose $i+2$ then the original trajectory can be replaced with $o_a^i \rightarrow o_a^{i+2} \rightarrow o_b^j \rightarrow o_b^{j+1}$.

These phases can be repeated, starting from Phase 1, to continually improve the path quality. Once the smoothing phases are complete, Phase 2 can be applied over the entire trajectory, resulting in the final smoothed solution. In practice,

a single-pass of this process and a final application of Phase 2 are sufficient.

V. PROPERTIES

Every pose along a valid edge in the RPG needs to be reachable from q_s^M . This requirement implies that the proposed method solves rearrangement problems that satisfy the following property: *there is a solution path for which every stable grasped pose along this path is reachable from q_s^M .*

The problems studied here are solved in a sequential manner, given a single arm, where the solution sequence can be decomposed into transfer and transit paths. Each transfer segment corresponds to $k-1$ objects remaining static and one object moved from a stable pose p to another p' . Such a motion can be discovered by the proposed approach if a $k+1$ -pose RPG is sampled, that contains the $k-1$ poses of the static objects, as well as p and p' . Since the solution path contains the motion $p' \rightarrow p'$, the RPG will also be able to discover this edge, as long as poses p and p' are reachable from q_s^M . So, in the worst case using RPGs of size $k+1$, this method needs to generate all the RPGs that correspond to each transfer motion of a solution trajectory. The RPGs that represent the solution motions can be pairwise connected, since they share k poses. Thereby, if all the RPGs corresponding to transfer paths of the solution are sampled, the edges that correspond to the motions within the RPGs are guaranteed to exist and the hyper-nodes belonging to the solution will be pairwise connected. The approach will discover such a solution, if it exists, and hence it is *probabilistically complete* within the set of problems that satisfy the assumption about q_s^M .

In practice, higher values of “blank” poses b than 1, provide computational benefits as a higher number of arrangements can be represented by a single RPG. There is a trade-off, however, since as b increases, the connectivity of the RPG will be decreasing. The experimental section evaluates this trade-off.

The path planner used for computing the paths of the manipulator is assumed to be complete in the above discussion. The following section provides a way to compute such paths, given a pre-computed manipulation roadmap, implying a dependence

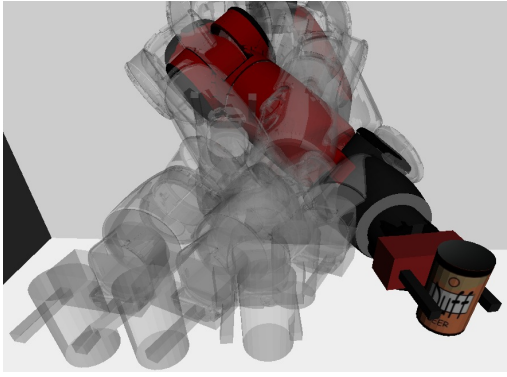


Fig. 6. Transition states $x^t \in X^t$, i.e., stable grasping configurations. An IK solution for the end effector is computed.

of the overall method on the probabilistic completeness of that approach.

VI. EFFICIENT COMPUTATIONS FOR MANIPULATION PATHS

A manipulation planning primitive is needed during the construction of the RPGs to detect whether an edge on the RPG can be added. For the efficient online computation of this operation, this work generates a manipulation roadmap $\mathcal{R}(\mathcal{V}, \mathcal{E})$ offline, for a single object and the static geometry. In this manner, the roadmap operates over reduced collision-free single-object states of the form $x = ((q^M, p), t)$, where $q^M \in \mathcal{Q}^M$ is a manipulator configuration, p is the pose of the single object and t is the type, i.e., either a transfer or a transit state.

The first step in the construction of \mathcal{R} is to sample transition states $x^t \in \mathbb{X}^t$, i.e., stable grasping configurations. This can be achieved by first sampling a stable collision-free object pose, $p \in \mathcal{P}^s$, and then using inverse kinematics to define the corresponding manipulator’s grasping configuration q^M (Fig. 6). If the inverse kinematics function returns a solution and the resulting configuration (q^M, p) is collision-free, then two states are generated with the same configuration: one of the transit type and one of the transfer type. Both of these states are inserted in \mathcal{R} and an edge is added between them.

In the experiments performed for this paper multiple attempts to generate grasping configurations for the manipulator take place for an individual object pose. Furthermore, two extra states are inserted in \mathcal{R} during this process. A transit state, called a “retraction” state, where the arm’s end-effector is slightly retracted from the object, and a transfer state, called a “raised” state, where the arm’s end-effector is slightly raised with the object. These additional states help in the connectivity of both the transit and transfer subsets of \mathcal{R} .

Given the set of transition states and edges, the method proceeds to separately explore the transit and the transfer subset of the state space in a PRM* fashion. For the transit component, additional grasping configurations for the manipulator are sampled, where the object is no longer required to be in a stable pose. Then neighboring transit states according to a

configuration space metric are considered for connection by interpolating between the two configurations. If the interpolated path is collision-free, the edge is added to the roadmap. The same process is repeated for transfer states. The objective is to achieve a roadmap that has the minimum number of connected components, so that the manipulator is able to transfer objects among the sampled transition states.

After the roadmap is constructed offline, it can also be informed about collision between potential objects placed on stable poses and the arm moving along edges of \mathcal{R} . All the sampled poses that have been used to build the manipulation roadmap are checked for collisions with the edges of the roadmap. In this way, all the necessary collision-checking can take place offline. This information is used during the online process in order to speed up the search for the “minimum conflict path” during the construction of RPGs.

Fig. 7 describes the use of the roadmap \mathcal{R} for the online generation of RPGs. First the poses of the RPG are selected from the poses used by \mathcal{R} . The roadmap stores multiple configurations $q^M \in \mathcal{Q}^M$, which can grasp an object in a stable configuration for stable pose. The safe configuration q_s^M is also part of the roadmap \mathcal{R} . To add an edge to the RPG between poses p and p' , the three segments path is evaluated on the roadmap. A heuristic search approach for “minimum conflict path-planning” on the roadmap is used for this process in order to find the least constrained path, $q_s^M \rightarrow x_p^t \rightarrow x_{p'}^t \rightarrow q_s^M$ given the other poses.

VII. EVALUATION

The proposed algorithm has been evaluated in a simulation environment to determine its scalability and to showcase the class of difficult non-monotone problems it can solve. The model of the manipulator used corresponds to a 7-DOF left arm of a Baxter. The solution paths were executed on a Baxter robot in open loop trials. The identical objects are designed as cylinders with height 14cm and radius 4cm. The class of non-monotone problems (needing multiple grasps of an object), which the algorithm addresses, have not been solved by any efficient method to the authors knowledge. Comparisons with the brute-force exploration has not been performed; a search on the combined configuration space of the manipulator and the objects is computationally inefficient.

To evaluate the proposed approach, a series of rearrangement problems were considered. These problems include environments with random initial configurations to a final grid rearrangement of the objects in a shelf, as well as two non-monotone problems with $k = 3, 4$ (Figure 8). The execution time and the length of the solution trajectory have been analyzed. These metrics are reported for different values of b , i.e., the number of “blank” poses in the RPGs. An informed pre-computed roadmap \mathcal{R} as described in section VI has been used for the experiments. A trial is a failure if the method cannot find a solution within the time limit of 600 seconds. The simulations were performed on a machine running on an Intel(R) Xeon(R) CPU E5-4650 0 @ 2.70GHz.

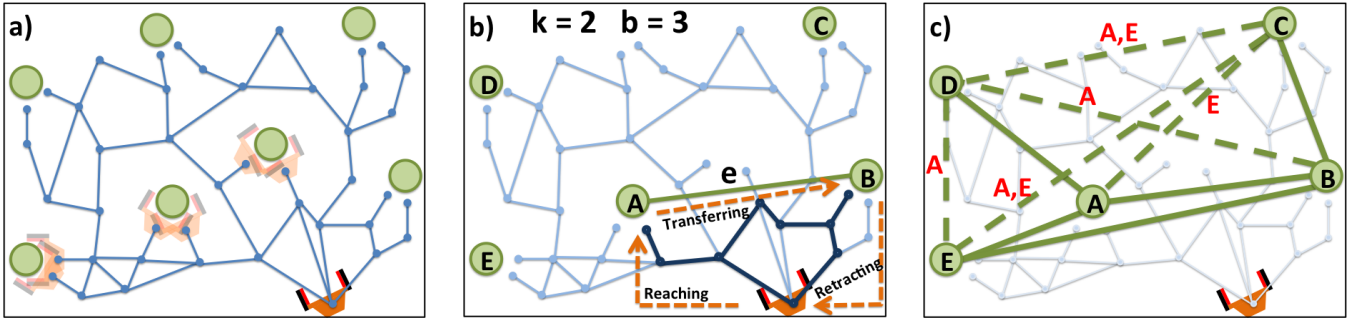


Fig. 7. RPG creation using a manipulator roadmap \mathcal{R} : a) The algorithm selects $k + b$ stable poses from \mathcal{R} to generates a pumped arrangement. b) An edge is added if it is possible given \mathcal{R} to transfer an object between these poses (A to B) without collisions given that all other poses are potentially occupied. The method identifies if the “minimum conflict path” has a constraint. c) The new RPG is returned with the collision free edges and the constrained edges in a separate structure E_c .

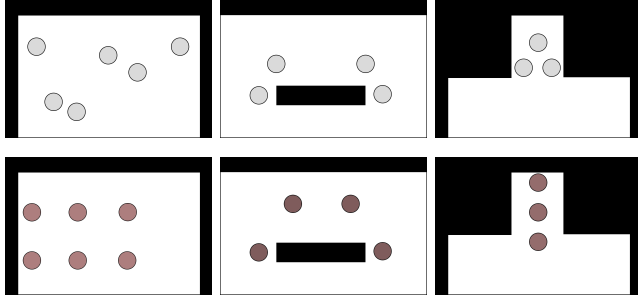


Fig. 8. The top and bottom rows are the initial and final positions for the randomized grid problem(left), the non-monotone benchmark 1(middle), the non-monotone benchmark 2(right)

Randomized grid evaluation: The first set of experiments populates a shelf like environment with sets of 2, 3, 4, 6, or 8 objects in mutually exclusive start poses. The goal arrangement is a uniform grid formation on the shelf. The dimensions of the shelf prevent the grasping of the objects from the top. This causes objects to be occluded by other objects in front of them and makes the rearrangement challenge non-trivial. Different values of $b = 1, 2, 3, 4$ used for these set of experiments. The manipulation roadmap used for these experiments used 20 different poses and got 517 vertices and 6032 edges.

Non-monotone benchmarks: A second set of experiments with 2 non-monotone benchmark problems is used to evaluate the method. For the first problem 4 objects are placed on a platform which resembles the shelf with the sides removed. The goal arrangement requires the two front objects to be moved at least twice. The trials are performed for different values of $b = 1, 2, 3, 4$. The same roadmap as before is used.

For the second benchmark, 3 objects are placed on a shelf with two static obstacles that form a narrow cavity between them. Because of the smaller k only values of $b = 1, 2, 3$ are tested. A different roadmap is used because of the constrained nature of the problem. This roadmap is computed with 30 poses and constructed with 684 vertices, 9628 edges.

A. Execution time

Fig.9(top) shows that the algorithm achieves a high success ratio for $k = 2, 3, 4, 6$. Trials with 8 objects could not finish within the stipulated limit of 600s. $k = 8$ causes an increase in the problem complexity. The case with 8 objects is constrained by the size of the shelf in terms of the availability of free poses on the shelf and free volume required to move. This affects the motion planning complexity for pose connections. The relatively sparse roadmap used in the experiment, helps the running time of the problems, but for the constraining setup, the solution is not always achieved within the time limit.

The value of b seems to have a significant role in the execution time for the same k . $b = 1$ seems to deteriorate the algorithm’s running time for higher values of k . This is a result of a very slow rate of exploration of the poses available on the shelf for rearrangement, due to only one empty pose available in the RPG. However the connectivity in an RPG is maximized. High values of b introduce a high combinatorial component of k objects in $k + b$ poses in every RPG and the connectivity of the RPG decreases. For every value of k the value of b with the best performance in terms of execution time, indicates this trade-off.

B. Solution quality

The quality of the solution in the randomized grid experiment, shown in Fig.9(bottom). The value of $b = 1$ introduces a low exploration rate, which increases the length of the solution. The conservative solution trajectories are shortened by an average of 48%.

C. Connectivity

The key feature of the algorithm is the way it constructs the hyper-graph from the hyper-nodes. The connectivity of this hyper-graph is crucial in finding the solution to the rearrangement problem. Fig.10(right) shows the rate at which new edges are created, with the introduction of every new hyper-node, for the different trials for the randomized grid experiment with $k = 6, b = 1$. The random trials corresponding to the best values of b for every $k = 2, 3, 4, 6$ from Fig.9 are analyzed for the number of hyper-nodes that were required

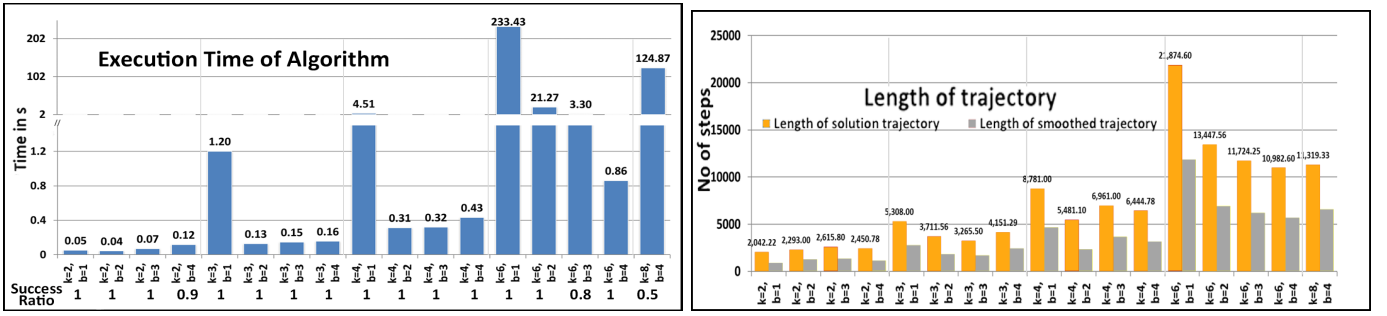
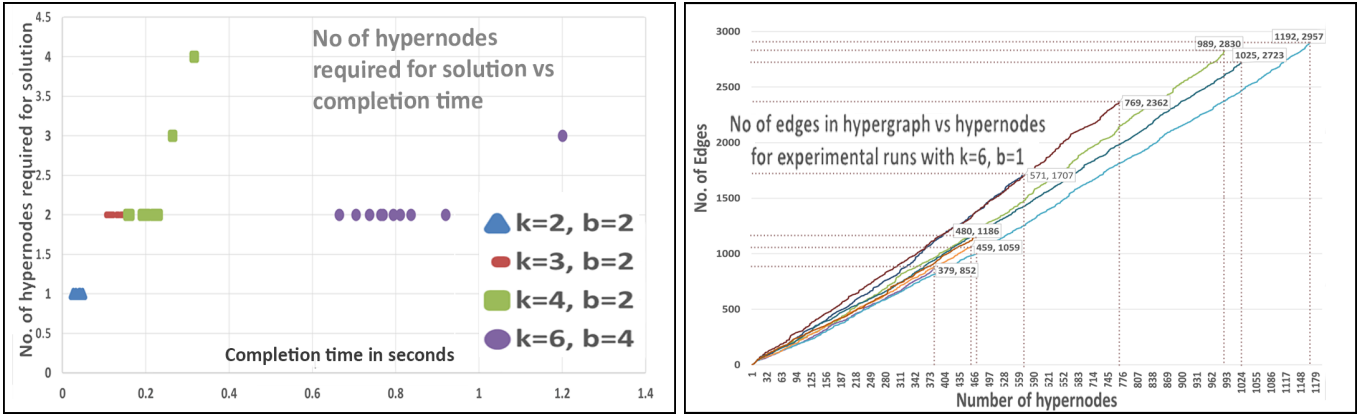


Fig. 9. (Top) Average running time for the algorithm to find a solution to the randomized grid problem for different values of k and b . The bottom row indicates success ratio over 10 trials. (Bottom) Average length of the trajectory before and after smoothing for the randomized grid evaluation and different values of k and b .



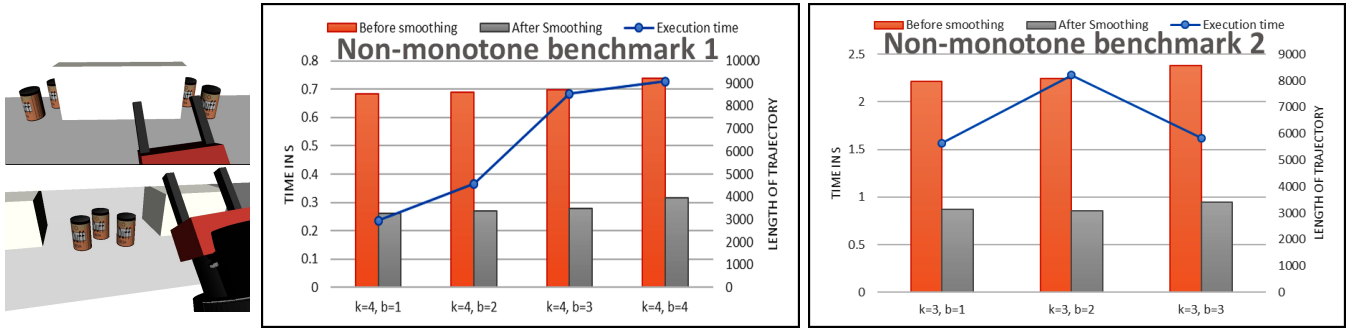


Fig. 11. The simulation environment on the left are for benchmark 1(top) and the benchmark 2(bottom). The performance of the algorithm for the non-monotone benchmark 1(middle) and non-monotone benchmark 2(right) are also shown.

computation of robust rearrangement trajectories and their robust execution given appropriate sensing input [32].

REFERENCES

- [1] R. Alami, T. Siméon, and J.-P. Laumond. A Geometrical Approach to Planning Manipulation Tasks. In *ISRR*, pages 113–119, 1989.
- [2] R. Alami, J.-P. Laumond, and T. Siméon. Two Manipulation Planning Algorithms. In *WAFR*, 1996.
- [3] B. Aronov, M. de Berg, A. F. van den Stappen, P. Svestka, and J. Vleugels. Motion Planning for Multiple Robots. *Discrete and Computational Geometry*, 1999.
- [4] V. Auletta, A. Monti, D. Parente, and G. Persiano. A Linear Time Algorithm for the Feasibility of Pebble Motion on Trees. *Algorithmica*, 23:223–245, 1999.
- [5] O. Ben-Shahar and E. Rivlin. Practical Pushing Planning for Rearrangement Tasks. *IEEE TRA*, 14(4), August 1998.
- [6] D. Berenson, R. Diankov, K. Nishiwaki, S. Kagami, and J. J. Kuffner. Grasp Planning in Complex Scenes. In *Proc. of IEEE-RAS International Conference on Humanoid Robots*, 2007.
- [7] D. Berenson, S. S. Srinivasa, D. Ferguson, and J. J. Kuffner. Manipulation Planning on Constraint Manifolds. In *ICRA*, 2009.
- [8] D. Berenson, S. S. Srinivasa, and J. J. Kuffner. Task Space Regions: A Framework for Pose-Constrained Manipulation Planning. *IJRR*, 30(12):1435–1460, 2012.
- [9] T. Bretl, S. Lall, J.-C. Latombe, and S. Rock. Multi-step Motion Planning for Free-Climbing Robots. In *WAFR*, 2004.
- [10] G. Calinescu, A. Dumitrescu, and J. Pach. Reconfigurations in Graphs and Grids. *SIAM Journal on Discrete Mathematics*, 22(1):124–138, 2008.
- [11] S. Cambon, R. Alami, and F. Gravot. A Hybrid Approach to Intricate Motion, Manipulation, and Task Planning. *IJRR*, (28), 2009.
- [12] P. C. Chen and Y. K. Hwang. Practical Path Planning Among Movable Obstacles. In *ICRA*, pages 444–449, 1991.
- [13] M. Ciocarlie, K. Hsiao, G. E. Jones, S. Chitta, R. B. Rusu, and I. A. Sucan. Towards Reliable Grasping and Manipulation in Household Environments. In *International Symposium on Experimental Robotics (ISER)*, 2010.
- [14] J. B. Cohen, S. Chitta, and M. Likhachev. Search-based Planning for Manipulation with Motion Primitives. In *ICRA*, 2010.
- [15] A. Cosgun, T. Hermans, V. Emeli, and M. Stilman. Push Planning for Object Placement on Cluttered Table Surfaces. In *IROS*, 2011.
- [16] E. Demaine, J. O’Rourke, and M. L. Demaine. Pushpush and push-1 are NP-hard in 2D. In *Canadian Conf. on Computational Geometry*, pages 211–219, 2000.
- [17] M. Dogar and S. Srinivasa. A Push-Grasping Framework in Clutter. In *RSS*, 2011.
- [18] G. Goral and R. Hassin. Multi-Color Pebble Motion on Graphs. *Algorithmica*, 58(3):610–636, 2010.
- [19] D. Halperin, J.-C. Latombe, and R. H. Wilson. A General Framework for Assembly Planning: the Motion Space Approach. *Algorithmica*, 26(3-4):577–601, 2000.
- [20] K. Hauser. The Minimum Constraint Removal Problem with Robotics Applications. In *WAFR*, 2012.
- [21] K. Hauser. Minimum Constraint Displacement Motion Planning. In *RSS*, 2013.
- [22] K. Hauser and J.-C. Latombe. Multi-Modal Planning in Non-Expansive Spaces. *IJRR*, 29(7):897–915, 2010.
- [23] K. Hauser and V. Ng-Thow-Hing. Randomized Multi-Modal Motion Planning for a Humanoid Robot Manipulation Task. *IJRR*, 2011.
- [24] D. Hsu, R. Kindel, J. C. Latombe, and S. Rock. Randomized Kinodynamic Motion Planning with Moving Obstacles. *IJRR*, 21(3):233–255, March 2002.
- [25] L.P. Kaelbling and T. Lozano-Pérez. Integrated Robot Task and Motion Planning in the Now. *CSAIL Technical Report*, 2012.
- [26] S. Karaman and E. Frazzoli. Sampling-based Algorithms for Optimal Motion Planning. *IJRR*, 30(7):846–894, June 2011.
- [27] L. E. Kavraki, P. Svestka, J.-C. Latombe, and M. Overmars. Probabilistic Roadmaps for Path Planning in High-Dim. Configuration Spaces. *IEEE TRA*, 1996.
- [28] J. King, M. Klingensmith, C. Dellin, M. Dogar, P. Velagapudi, N. Pollard, and S. S. Srinivasa. Pregrasp Manipulation as Trajectory Optimization. In *RSS*, 2013.
- [29] D. Kornhauser, G. Miller, and P. Spirakis. Coordinating Pebble Motion on Graphs, the Diameter of Permutation Groups, and Applications. In *FOCS*, 1984.
- [30] S. M. LaValle and J. J. Kuffner. Randomized Kinodynamic Planning. *IJRR*, 2001.
- [31] S. Leroy, J.-P. Laumond, and T. Siméon. Multiple Path Coordination for Mobile Robots: A Geometric Algorithm. In *IJCAI*, 1999.
- [32] M. LeVinh, J. Scholz, and M. Stilman. Hierarchical Decision Theoretic Planning for Navigation Among Movable Obstacles. In *WAFR*, 2012.
- [33] R. Luna and K. E. Bekris. Efficient and Complete Centralized Multi-Robot Path Planning. In *IROS*, 2011.
- [34] P. Mike, V. Hwang, S. Chitta, and M. Likhachev. Learning to Plan for Constrained Manipulation from Demonstrations. In *RSS*, 2013.
- [35] D. Nieuwenhuisen, A. Frank van der Stappen, and M. H. Overmars. An Effective Framework for Path Planning amidst Movable Obstacles. In *WAFR*, 2006.
- [36] J. Ota. Rearrangement Planning of Multiple Movable Objects. In *ICRA*, 2004.
- [37] T. Siméon, J.-P. Laumond, J. Cortés, and A. Sahbani. Manipulation Planning with Probabilistic Roadmaps. *IJRR*, (23), 2004.

- [38] K. Solovey and D. Halperin. k-Color Multi-Robot Motion Planning. In *WAFR*, 2012.
- [39] M. Stilman and J. Kuffner. Navigation among Movable Obstacles: Realtime Reasoning in Complex Environments. In *Humanoid Robotics*, pages 322–341, 2004.
- [40] M. Stilman and J. J. Kuffner. Planning Among Movable Obstacles with Artificial Constraints. In *WAFR*, 2006.
- [41] M. Stilman, J. Schamburek, J. J.J. Kuffner, and T. Asfour. Manipulation Planning Among Movable Obstacles. In *ICRA*, 2007.
- [42] S. Sundaram, I. Remmler, and N. M. Amato. Disassembly Sequencing Using a Motion Planning Approach. In *ICRA*, pages 1475–1480, 2001.
- [43] J. van den Berg and M. Overmars. Prioritized Motion Planning for Multiple Robots. In *IROS*, pages 430–435, 2005.
- [44] J. van den Berg, M. Stilman, J. J. Kuffner, M. Lin, and D. Manocha. Path Planning Among Movable Obstacles: A Prob. Complete Approach. In *WAFR*, 2008.
- [45] J. van den Berg, J. Snoeyink, M. Lin, and D. Manocha. Centralized Path Planning for Multiple Robots: Optimal Decoupling into Sequential Plans. In *RSS*, 2009.
- [46] G. Wagner, M. Kang, and H. Choset. Probabilistic Path Planning for Multiple Robots with Subdimensional Expansion. In *ICRA*, 2012.
- [47] G. Wilfong. Motion Planning in the Presence of Movable Obstacles. In *Annual Symp. of Computational Geometry*, pages 279–288, 1988.
- [48] R. H. Wilson and J.-C. Latombe. Geometric Reasoning about Mechanical Assembly. *Artificial Intelligence Journal*, 71(2): 371–396, 1994.
- [49] J. Yu and S. M. LaValle. Multi-agent Planning and Network Flow. In *WAFR*, 2012.
- [50] M. Zucker, N. Ratliff, A. Dragan, M. Pivtoraiko, M. Klingensmith, C. Dellin, J. A. Bagnell, and S. S. Srinivasa. CHOMP: Covariant Hamiltonian Optimization for Motion Planning. *IJRR*, 2013.